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# A model for $F_L$ and $R = F_L/F_T$ at low $x$ and low $Q^2$

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## Abstract

A model for the longitudinal structure function  $F_L$  at low  $x$  and low  $Q^2$  is presented, which includes the kinematical constraint  $F_L \sim Q^4$  as  $Q^2 \rightarrow 0$ . It is based on the photon-gluon fusion mechanism suitably extrapolated to the region of low  $Q^2$ . The contribution of quarks having limited transverse momentum is treated phenomenologically assuming that it is described by the soft pomeron exchange mechanism. The ratio  $R = F_L/(F_2 - F_L)$ , with the  $F_2$  appropriately extrapolated to the region of low  $Q^2$ , is also discussed.

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# 1 Introduction

The longitudinal structure function  $F_L(x, Q^2)$  which corresponds to the interaction of the longitudinally polarized virtual photon in the one-photon-exchange mechanism of inelastic lepton-nucleon scattering is a very interesting dynamical quantity since at least at low  $x$  the dominant contribution to it comes from gluons. The experimental determination of  $F_L$  is in general difficult and requires a measurement of the  $y$  dependence of the deep inelastic cross section for fixed  $x$  and  $Q^2$  where, as usual,  $y = (p_l p)/(q p)$ ,  $x = Q^2/(2pq)$  and  $Q^2 = -q^2$  with  $p_l$ ,  $p$  and  $q$  denoting the four momentum of the incident lepton, the four momentum of the proton and four momentum transfer between the leptons respectively, see fig. 1. In the "naive" quark-parton model the structure function  $F_L(x, Q^2)$  vanishes. To be precise, in the limit of large  $Q^2$  it is proportional to  $(\langle m^2 \rangle + \langle \kappa_T^2 \rangle)/Q^2$  where  $m$  is the quark mass and  $\kappa_T$  is the transverse momentum of the quark which, in the naive parton model, is by definition limited [1, 2, 3]. This remains approximately valid in the leading logarithmic approximation where only collinear gluon emissions are considered. The limitation of the quark transverse momentum no longer holds in the next-to-leading logarithmic approximation when the point-like QCD interactions allow (with probability  $\sim \alpha_s(Q^2)$ )  $\langle \kappa_T^2 \rangle$  to grow as  $Q^2$  with increasing  $Q^2$ . Thus  $F_L(x, Q^2)$  acquires a leading twist contribution, proportional to  $\alpha_s(Q^2)$  [4, 5, 6]. At small  $x$  it is driven by the gluons through the  $g \rightarrow q\bar{q}$  transition and, in fact,  $F_L$  can be used as a very useful quantity for a direct measurement of the gluon distribution in a nucleon [7].

In the limit  $Q^2 \rightarrow 0$  the structure function  $F_L$  has to vanish as  $Q^4$  (for fixed  $2pq$ ). This kinematical constraint eliminates potential kinematical singularities at  $Q^2 = 0$  of the hadronic tensor  $W^{\mu\nu}$  defining the virtual Compton scattering amplitude [8]. It also reflects the simple physical fact that the total cross section  $\sigma_L \sim F_L/Q^2$  describing the interaction of the longitudinally polarized virtual photons has to vanish in the real photoproduction limit.

The differential cross section for inelastic lepton-nucleon scattering is most frequently expressed in terms of the structure function  $F_2(x, Q^2)$  and the ratio  $R(x, Q^2) = F_L(x, Q^2)/F_T(x, Q^2)$  where  $F_T(x, Q^2) = F_2(x, Q^2) - F_L(x, Q^2)$  is the structure function associated with transversely polarized virtual photons. Unfortunately measurements of  $R(x, Q^2)$  are difficult and thus scarce, especially at low  $x$  and/or low  $Q^2$ . For  $x < 0.01$  and/or  $Q^2 < 0.5$  GeV $^2$  the experimental information is particularly poor. The measurements of  $R$  are compiled in fig.2. Preliminary measurements in the low  $x$  region have been announced by the CCFR collaboration [15] and by NMC [16]. All the data indicate a small value of  $R$  at moderate values of  $x$  and an increase of  $R$  with decreasing  $x$ . The data collected in fig.2 come from experiments carried out on different targets. This is justified since the measured differences  $R^A - R^d$  do not exhibit any significant dependence on  $x$  and they are consistent with zero, [17]. All the data in fig.2 (except those of SLAC E140X) have been fitted to a common function (the curve in fig.2),  $R^{fit}$ , which is constructed to extrapolate to theoretically reasonable values outside the kinematical range of the data: namely at  $x \rightarrow 0$ ,  $x \rightarrow 1$  and  $Q^2 \rightarrow \infty$ . In particular the last constraint means that  $R^{fit}$  matches  $R^{QCD}$ , [4], at high  $Q^2$ . The fit expression should not however be used for

$Q^2 < 0.35 \text{ GeV}^2$  which means that no constraints are given on the behaviour of  $R$  at low  $Q^2$ . This is a serious deficiency in the procedure used to extract polarized and non-polarized structure functions from the experimental data, especially in the calculation of radiative corrections. Thus the procedure commonly accepted in data analysis is to assume  $R(x, Q^2) = R(x, Q^2 = 0.35 \text{ GeV}^2)$  with 100% uncertainty for  $Q^2 < 0.35 \text{ GeV}^2$ . The uncertainty of  $R$  is then propagated into a contribution (usually the dominant one) to the systematic error on the extracted structure functions. The need for a more precise determination of  $R$  at low  $Q^2$  is thus clearly seen.

At large  $Q^2$  and large  $x$  ( $x > 0.1$ ) perturbative QCD describes reasonably well the available data on the ratio  $R(x, Q^2)$  [18]. It has however been noticed that in order to correctly accommodate experimental information from the moderately large  $Q^2$  region, a significant "higher twist" contribution, which vanishes as  $1/Q^2$  at large  $Q^2$ , has to be added as well [15, 19, 20].

While the longitudinal structure function is (at least theoretically) fairly well understood at high  $Q^2$  very little (if anything) is known about its possible extrapolation towards the region of low  $Q^2$ . This should be contrasted with the structure function  $F_2(x, Q^2)$  for which several dynamically motivated extrapolations exist, cf. [8].

The purpose of this paper is to provide an extrapolation of the structure function  $F_L(x, Q^2)$  towards the region of moderate and low values of  $Q^2$ . This extrapolation will be based on the photon-gluon fusion mechanism, essential at low  $x$  and suitably extended into the region of low  $Q^2$ . A similar description of both the structure functions  $F_2$  and  $F_L$  has been considered in ref. [21]. We shall explicitly introduce an additional higher twist term which comes from the contribution of the quarks (antiquarks) with limited transverse momentum. We assume that this term is described by "soft" pomeron exchange. We treat this contribution phenomenologically and determine its normalisation from the (non-perturbative) part of the structure function  $F_2(x, Q^2)$ . The model embodies the kinematical constraint  $F_L \sim Q^4$  in the limit  $Q^2 \rightarrow 0$  (for fixed  $2pq$  or  $\nu$ ).

The paper is organised as follows: in the next section we discuss the photon-gluon fusion contribution to the structure function  $F_L$  and its extension to the low  $Q^2$  region. In section 3 we present a phenomenological model for the higher twist contribution to  $F_L$  at small  $x$  which is based on the soft pomeron exchange mechanism. Section 4 contains numerical predictions for  $F_L$  and  $R$ , while section 5 contains a brief summary of our results.

## 2 The photon - gluon fusion model for the structure function $F_L$

At small  $x$  and at large  $Q^2$  the structure functions can be computed from the  $k_T$  factorization theorem [22, 23, 24] which corresponds to the (virtual) photon - gluon fusion mechanism shown

in fig.3. The structure function  $F_L$  obtained from the diagram of fig.3 is given by [25] :

$$F_L(x, Q^2) = 2 \frac{Q^4}{\pi^2} \sum_q e_q^2 I_q(x, Q^2) \quad (1)$$

where,

$$I_q(x, Q^2) = \int \frac{dk_T^2}{k_T^4} \int_0^1 d\beta \int d^2 \kappa'_T \alpha_s \beta^2 (1-\beta)^2 \frac{1}{2} \left( \frac{1}{D_{1q}} - \frac{1}{D_{2q}} \right)^2 f(y_q, k_T^2) \quad (2)$$

and

$$\begin{aligned} D_{1q} &= \kappa_T^2 + \beta(1-\beta)Q^2 + m_q^2 \\ D_{2q} &= (\kappa_T - \mathbf{k}_T)^2 + \beta(1-\beta)Q^2 + m_q^2. \end{aligned} \quad (3)$$

$\alpha_s$  is the QCD coupling and its argument will be specified later. The transverse momentum  $\kappa'_T$  and the variables  $y_q$  are defined as follows:

$$\kappa'_T = \kappa_T - (1-\beta)\mathbf{k}_T \quad (4)$$

$$y_q = x \left( 1 + \frac{\kappa_T'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k_T^2}{Q^2} \right) \quad (5)$$

where  $\kappa_T$  is the transverse momentum of the quark(antiquark) in the quark box (see fig.3). The variable  $\beta$  is the corresponding Sudakov parameter appearing in the quark momentum decomposition into the basic lightlike four vectors,  $p'$  and  $q'$ ,

$$\begin{aligned} \kappa &= x_q p' - \beta q' + \kappa_T \\ x_q &= x \left( 1 + \frac{m_q^2 + \kappa_T^2}{(1-\beta)Q^2} \right) \\ p' &= p - \frac{M^2 x}{Q^2} q \\ q' &= q + x p \end{aligned} \quad (6)$$

where  $M$  denotes the nucleon mass. The function  $f(y, k^2)$  is the so – called unintegrated gluon distribution. It is related to the (scale dependent) gluon distribution  $g(y, \mu^2)$  by

$$y g(y, \mu^2) = \int^{\mu^2} \frac{dk_T^2}{k_T^2} f(y, k_T^2). \quad (7)$$

The parameters  $m_q$  are the masses of the quarks. It should be noted that the integrals  $I_q$  defined by (2) are infrared finite even if we set  $m_q = 0$ . The non-zero values of the quark masses are however necessary if formula (1) is extrapolated down to  $Q^2 = 0$  respecting the kinematical constraint  $F_L \sim Q^4$ . The non-zero quark masses then play the role of the infrared regulator. It should be noticed that the expressions (1,2) define  $F_L$  as an analytic function of  $Q^2$  except for the cut  $Q^2 < -4m_q^2$ . Equation (2) can be interpreted as representing the rescattering of the  $q\bar{q}$  fluctuation of the virtual photon through the exchange of two and more

interacting gluons. Both the diagonal and non-diagonal transitions in this rescattering are included. We assume that equations (1,2) embody (at least in the average sense and as far as the position of the singularities in the  $Q^2$  plane are concerned) also the virtual vector meson contributions which couple to virtual photons. This will fix the magnitude of the corresponding quark mass parameters to be  $m_q^2 \approx m_v^2/4$  where  $m_v$  denotes the mass of the lightest vector meson corresponding to the  $q\bar{q}$  pair.

The integration limits in (2) are constrained by the condition:

$$y_q < 1 \quad (8)$$

The condition  $y_q > x$  is automatically satisfied (see eq.(5) )

The unintegrated gluon distribution  $f(x, k^2)$  should in principle be obtained from the solution of the Balitzkij–Fadin–Kuraev–Lipatov (BFKL) equation [26, 27]. In our calculations we shall use the following approximation,

$$f(y, k_T^2) = y \frac{\partial g^{AP}(y, Q^2)}{\partial \ln Q^2} \Big|_{Q^2=k_T^2} \quad (9)$$

where  $g^{AP}(y, Q^2)$  satisfies the conventional (LO or NLO) Altarelli-Parisi equations. In this approximation one neglects the higher order small  $x$  resummation effects in the gluon anomalous dimension. The gluon anomalous dimension  $\gamma_{gg}(\alpha_s, \omega)$  is the moment of the splitting function  $P_{gg}(z)$ ,

$$\gamma_{gg}(\alpha_s, \omega) = \int_0^1 dz z^\omega P_{gg}(z, \alpha_s) . \quad (10)$$

The BFKL equation effectively resums the leading powers of  $\alpha_s/\omega$  i.e.

$$\gamma_{gg}(\alpha_s, \omega) = \frac{\bar{\alpha}_s}{\omega} + \sum_{n=4} D_n \left( \frac{\bar{\alpha}_s}{\omega} \right)^n \quad (11)$$

( $\bar{\alpha}_s = 3\alpha_s/\pi$ ), where the coefficients  $D_n$  can be calculated from the kernel of the BFKL equation [28]. It should be noted that the sum on the right-hand-side of (11) only starts at  $n = 4$ . As a result it affects the evolution of gluon distribution (and of observable quantities) only at very small values of  $x$  ( $x < 10^{-4}$ ). In our approximation this contribution is neglected.

Expression (1) can be represented in the following "  $k_t$  factorized" form:

$$F_L(x, Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2} \hat{F}_L^0(x', Q^2, k_T^2) f\left(\frac{x}{x'}, k_T^2\right) \quad (12)$$

where the function  $\hat{F}_L^0(x', Q^2, k_T^2)$  can be regarded as the longitudinal structure function of the off-shell gluon with virtuality  $k_T^2$ . It corresponds to the quark box (and crossed-box) diagrams in the upper part of the diagram in fig.3 and is given by the following formula:

$$\hat{F}_L^0(x', Q^2, k_T^2) = \frac{Q^4}{\pi^2 k_T^2} \sum_q e_q^2 \int_0^1 d\beta \int d^2 \kappa_T' x' \delta \left( x' - \left( 1 + \frac{\kappa_T'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k_T^2}{Q^2} \right)^{-1} \right) \times \quad (13)$$

$$\times \alpha_s \beta^2 (1 - \beta)^2 \left( \frac{1}{D_{1q}} - \frac{1}{D_{2q}} \right)^2 \quad (14)$$

where  $D_{1q}, D_{2q}$  are defined by equation (3). The leading twist part of the  $k_t$  factorization formula can be rewritten in a collinear factorized form:

$$F_L(x, Q^2) = \int_x^1 \frac{dx'}{x'} C_L(x', Q^2, \alpha_s(Q^2)) \frac{x'}{x} g\left(\frac{x'}{x}, Q^2\right) \quad (15)$$

where the leading powers of  $\alpha_s \ln(x')$ ,  $(\alpha_s \ln(x/x'))$  have been resummed in  $C_L(x', Q^2, \bar{\alpha}_s(Q^2))$ ,  $(x/x' g(x/x', Q^2))$ . Thus for instance the moment function

$$\bar{C}_L(\omega, Q^2, \alpha_s(Q^2)) \equiv \int_0^1 \frac{dz}{z} z^\omega C_L(z, Q^2, \alpha_s(Q^2)) \quad (16)$$

is related to the moment function  $\tilde{F}_L^0(\omega, \gamma)$ , by

$$\bar{C}_L(\omega, Q^2, \alpha_s(Q^2)) = \gamma_{gg}(\alpha_s, \omega) \tilde{F}_L^0(\omega, \gamma = \gamma_{gg}(\alpha_s, \omega)) , \quad (17)$$

where

$$\begin{aligned} \bar{F}_L^0(\omega, Q^2) &= \frac{1}{2\pi i} \int_{1/2-i\infty}^{1/2+i\infty} d\gamma \tilde{F}_L^0(\omega, \gamma) \left( \frac{Q^2}{k_T^2} \right)^\gamma \\ \bar{F}_L^0(\omega, Q^2) &= \int_0^1 \frac{dx}{x} x^\omega \hat{F}_L^0(x, Q^2) . \end{aligned} \quad (18)$$

Our approximation corresponds to setting  $\gamma_{gg} = \bar{\alpha}_s/\omega$ . It correctly generates the first three corrections to  $C_L$  which are presumably most important at moderately small values of  $x$ .

In the "on-shell" approximation one sets  $k_T^2 = 0$  in the argument of  $\hat{F}_L^0(x', Q^2, k_T^2)$  and restricts the integration over  $k_T^2$  to the region  $k_T^2 \ll \kappa_T'^2 \sim Q^2$ . After taking into account (7) the structure function  $F_L$  can be expressed in terms of the conventional gluon distribution  $yg(y, Q^2)$ ,

$$I_q(x, Q^2) = \pi \int_0^1 d\beta \int d\kappa_T'^2 \alpha_s(Q^2) \beta^2 (1 - \beta)^2 \frac{\kappa_T'^2}{D_q^4} y_q g(y_q, Q^2) \quad (19)$$

where now

$$y_q = x \left( 1 + \frac{\kappa_T'^2 + m_q^2}{\beta(1 - \beta)Q^2} \right) \quad (20)$$

and

$$D_q = \kappa_T'^2 + \beta(1 - \beta)Q^2 + m_q^2 . \quad (21)$$

In order to extrapolate the formula (1) to the low  $Q^2$  region (with  $I_q$  given by (2) or (19)) we have to freeze the evolution of  $g(y, Q^2)$  and freeze the argument of  $\alpha_s(Q^2)$ . To this end we substitute  $Q^2 + 4m_q^2$  instead of  $Q^2$  as the argument of  $\alpha_s$  and of  $yg(y, Q^2)$ . This substitution may be justified by analyticity arguments i.e. we want  $I_q$  to have "threshold" singularities for  $Q^2 < -4m_q^2$ . Moreover, for heavy quarks, it is the heavy mass squared and not  $Q^2$  which should control the scale of  $\alpha_s$  and of the parton distributions for moderately large values of  $Q^2$ .

It should be noted that after those modifications the structure function  $F_L$  can be continued down to  $Q^2 = 0$ , respecting the kinematical constraint (  $F_L \sim Q^4$  in the limit  $Q^2 \rightarrow 0$  ).

It follows from (19) that the structure function  $F_L$  in the on-shell approximation can be represented by

$$F_L = 2 \sum_q e_q^2 (J_q^{(1)} - 2 \frac{m_q^2}{Q^2} J_q^{(2)}) \quad (22)$$

where

$$J_q^{(1)} = \frac{\alpha_s}{\pi} \int_{\bar{x}_q}^1 \frac{dy}{y} \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}} yg(y, Q^2) \quad (23)$$

$$J_q^{(2)} = \frac{\alpha_s}{\pi} \int_{\bar{x}_q}^1 \frac{dy}{y} \left(\frac{x}{y}\right)^3 \ln \left( \frac{1 + \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}}{1 - \sqrt{1 - \frac{4m_q^2 x}{Q^2(y-x)}}} \right) yg(y, Q^2) \quad (24)$$

and

$$\bar{x}_q = x \left(1 + \frac{4m_q^2}{Q^2}\right). \quad (25)$$

It should be noted that in the large  $Q^2$  limit, the first term  $J_q^{(1)}$  becomes equal to the standard QCD formula describing the leading order contribution to  $F_L$  originating from the photon-gluon fusion [4, 6],

$$F_L(x, Q^2) = 2 \frac{\alpha_s}{\pi} \sum_q e_q^2 \int_x^1 \left(\frac{x}{y}\right)^2 \left(1 - \frac{x}{y}\right) yg(y, Q^2). \quad (26)$$

In the large  $Q^2$  region formula (22) with non-zero quark masses generates also power corrections enhanced by logarithmic factor present in the term  $J_q^{(2)}$ .

When analysing the  $k_T$  factorization formula it is convenient to use the "on-shell" approximation in the region  $k^2 < k_0^2$ , where  $k_0^2$  is a parameter of the order 1 GeV<sup>2</sup> and to leave the remaining part ( $k^2 > k_0^2$ ) unchanged. The first term is then given by (22), (23), (24) where gluon distribution is evaluated at the scale  $k_0^2$  instead of  $Q^2$  [24]. This gives the following representation for the structure function  $F_L$ :

$$F_L(x, Q^2) = F_{L0}(x, Q^2; k_{T0}^2) + 2 \frac{Q^4}{\pi^2} \sum_q e_q^2 \int \frac{dk_T^2}{k_T^4} \Theta(k_T^2 - k_{T0}^2) \int_0^1 d\beta \int d^2 \kappa'_T \alpha_s \beta^2 (1 - \beta)^2 \frac{1}{2} \left(\frac{1}{D_{1q}} - \frac{1}{D_{2q}}\right)^2 f(y_q, k_T^2) \quad (27)$$

where  $F_{L0}(x, Q^2; k_{T0}^2)$  is given by equations (22) – (24) with  $yg(y, k_{0T}^2)$  instead of  $yg(y, Q^2)$  in the integrals (23) and (24). The argument of  $\alpha_s$  in the formulae (23), (24) defining  $F_L(x, Q^2) = F_{L0}(x, Q^2; k_{T0}^2)$  and in the second term of (27) will be set equal to  $Q^2 + 4m_q^2$ .

### 3 A model for the higher twist contribution to $F_L$

The phenomenological description of the experimental data for the ratio  $R(x, Q^2)$  in the region of moderately large values of  $Q^2$  and large values of  $x$  ( $x > 0.1$ ) requires a significant higher twist contribution, i.e. the term which vanishes as  $1/Q^2$  for  $Q^2 \rightarrow \infty$  [15, 19, 20].

In this section we present the phenomenological model for  $F_L$  which takes into account the higher twist effect. The idea is to treat the contributions from the regions of low and high values of quark transverse momenta in different ways. That is we divide the integration over  $\kappa_T'^2$  into two parts: the region  $0 < \kappa_T'^2 < \kappa_{0T}'^2$ , and  $\kappa_T'^2 > \kappa_{0T}'^2$ , where  $\kappa_{0T}'^2$  is an arbitrary cut-off, chosen to be of the order of 1 GeV<sup>2</sup>.

We leave unchanged the contribution coming from the high  $\kappa_T'^2$  region whereas in the low  $\kappa_T'^2$  region, which is presumably dominated by the soft physics, we use the "on-shell" approximation and make the substitution:

$$\alpha_s z g(z, Q^2) \rightarrow A. \quad (28)$$

This gives the following representation of the higher twist contribution to  $F_L$ :

$$F_L^{HT} = 2A \sum_q e_q^2 \frac{Q^4}{\pi} \int_0^1 d\beta \beta^2 (1-\beta)^2 \int_0^{\kappa_{0T}'^2} d\kappa_T'^2 \frac{\kappa_T'^2}{D_q^4} \quad (29)$$

where  $D_q$  has been defined in (21). The constant  $A$  is not a free parameter. We estimate it from  $F_2$  assuming that the non-perturbative contribution to  $F_2$  in the scaling region also comes from the region of low values of  $\kappa_T'^2$  and is controlled by the same parameter. The term  $F_L^{HT}$  can be interpreted as representing the contribution of soft pomeron exchange with intercept equal to 1.

It should be noted that  $F_L^{HT}$  given by equation (29) will vanish as  $1/Q^2$  in the high  $Q^2$  limit (modulo logarithmically varying factors). We call it therefore a "higher twist contribution". It should be noticed that this term will also respect the kinematical constraint  $F_L \sim Q^4$  in the limit  $Q^2 \rightarrow 0$ .

The corresponding formula describing the photon - gluon contribution to the structure function  $F_T$  is

$$\begin{aligned} F_T(x, Q^2) = & 2 \sum_q e_q^2 \frac{Q^2}{4\pi} \alpha_s \int_0^1 d\beta \int d\kappa_T'^2 \frac{x}{x'} g\left(\frac{x}{x'}, Q^2\right) \times \\ & \times \left[ \frac{\beta^2 + (1-\beta)^2}{2} \left( \frac{1}{D_q^2} - \frac{2\kappa_T'^2}{D_q^3} + \frac{2\kappa_T'^2 \kappa_T'^2}{D_q^4} \right) + \frac{m_q^2 \kappa_T'^2}{D_q^4} \right]. \end{aligned} \quad (30)$$

We integrate this expression over the low  $\kappa_T'^2$  region ( $\kappa_T'^2 < \kappa_{0T}'^2$ ) after substituting  $\alpha_s z g \rightarrow A$  and retain its limiting value as  $Q^2 \rightarrow \infty$ . We identify this contribution to  $F_T$  with the "background" term  $F_2^{Bg}$  of ref.[29] i.e.:

$$F_2^{Bg} = A \times \frac{\sum_q e_q^2}{\pi} \int_0^\infty dt \int_0^{\kappa_{0T}'^2} d\kappa_T'^2 \left[ \frac{1}{2} \left( \frac{1}{D_q^2} - \frac{2\kappa_T'^2}{D_q^3} + \frac{2\kappa_T'^2 \kappa_T'^2}{D_q^4} \right) + \frac{m_q^2 \kappa_T'^2}{D_q^4} \right] \quad (31)$$

where now

$$D_q = m_q^2 + \kappa_T'^2 + t . \quad (32)$$

A possible weak  $x$  dependence of  $F_2^{Bg}$  is neglected and we set  $F_2^{Bg} = 0.4$ .

The complete structure function  $F_L$  is represented as  $F_L = F_L^{HT} + F_L^{LT}$  where  $F_L^{LT}$  is calculated from equations (22) – (24), or from equation (27) with the additional constraint  $\kappa_T'^2 > \kappa_{0T}'^2$  in order to avoid double counting.

## 4 Numerical results

In this section we present the numerical analysis of the structure function  $F_L$ . We shall also perform the analysis of the ratio  $R(x, Q^2)$  and compare it with the available data.

In our calculations we have included the contributions of the  $u, d, s, c$  quarks with masses 0.35, 0.35, 0.5, 1.5 GeV respectively. The parameter  $\kappa_{0T}'^2$  was set to be equal 0.8 GeV<sup>2</sup>. We have varied this parameter in the interval 0.8–1.5 GeV<sup>2</sup> and found that the results for  $R$  are not very sensitive to these changes, i.e. the ratio  $R$  does not change by more than 10%.

We have used the GRV [30] and MRS(A) [31] parton distributions both in the "on-shell" and "off-shell" approximation. In the case of the GRV parametrisation LO gluons and QCD coupling constant were used. In our calculations we froze the evolution in  $Q^2$  in the argument of  $\alpha_s(Q^2)$  and also in the function  $yg(y, Q^2)$  as explained before. Besides the dominant (at small  $x$ ) photon-gluon fusion  $\gamma g \rightarrow q\bar{q}$  contribution, we have also included in  $F_L$  a contribution from quarks taking into account threshold effects in the corresponding convolution integral i.e.

$$\Delta F_L(x, Q^2) = \sum_i e_i^2 \frac{\alpha_s(Q^2 + 4m_q^2)}{\pi} \frac{4}{3} \int_{\bar{x}_q}^1 \frac{dy}{y} \left( \frac{x}{y} \right)^2 [q_i(y, Q^2 + 4m_q^2) + \bar{q}_i(y, Q^2 + 4m_q^2)] . \quad (33)$$

where  $\bar{x}_q$  is given by (25).

In figures 4 and 5 we show our results for the longitudinal structure function calculated without an additional higher twist contribution.  $F_L$  is plotted as the function of  $Q^2$  for different values of  $x$ , and as a function of  $x$  for different values of  $Q^2$ . The theoretical curves reflect the different behaviour of the MRS(A) and GRV gluon distributions. The difference between the on-shell and off-shell approaches is in general small, particularly in the small  $Q^2$  region. It can be seen from fig.5, that the magnitude of  $F_L$  strongly decreases as  $Q^2 \rightarrow 0$ .

In figures 6 and 7 we show results which include the additional higher twist contribution. It can be seen from these plots that the resulting magnitude of the longitudinal structure function is now bigger in the low  $Q^2$  region than in the previous case. It should also be noted that the structure function  $F_L$  flattens as a function of  $x$  in the low  $x$  and low  $Q^2$  region. This is a direct consequence of the "soft pomeron" parametrization of the higher twist term.

We have also calculated the ratio  $R(x, Q^2) = F_L(x, Q^2)/(F_2(x, Q^2) - F_L(x, Q^2))$ . The structure function  $F_2$  was parametrized following the model of ref. [32]. In this model the structure function  $F_2$  is represented by

$$F_2(x, Q^2) = \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2} + \frac{Q^2}{Q^2 + Q_0^2} F_2^{AS}(\bar{x}, Q^2 + Q_0^2). \quad (34)$$

The first term describes the VMD contribution and the sum extends over the low mass vector mesons. The parameter  $\gamma_v$  can be estimated from the leptonic widths of the vector mesons,  $\sigma_v(s)$  is the total vector meson - proton cross section, and  $s = 2pq - Q^2 + M^2$  where  $M$  is the nucleon mass. The structure function  $F_2^{AS}(x, Q^2)$  is obtained from the QCD improved parton model. The variable  $\bar{x}$  is defined as

$$\bar{x} = x(1 + \frac{Q_0^2}{Q^2}). \quad (35)$$

In the case of the GRV parametrization only the light quarks contribution was included in  $F_2^{AS}$  in the formula (34). The charm quark contribution to  $F_2$  was calculated from the on-shell photon-gluon fusion.

In figures 8 and 9 we show our results for the ratio  $R(x, Q^2)$ . We see that this quantity is sensitive to the different parton distributions which were used in calculating both  $F_L$  and  $F_2$ . In particular the structure function  $F_2$  obtained from the formula (34) with  $F_2^{AS}$  calculated from the MRS(A) parametrization turns out to be steeper function of  $x$  in the low  $Q^2$  region than that when  $F_2^{AS}$  is calculated from the GRV partons. As a result the ratio  $R$  decreases with  $x$  as  $x \rightarrow 0$ . This effect is weaker if  $F_L$  does not contain the higher twist modification. The spread of the theoretical predictions can be used as an estimate of the corresponding errors caused by the uncertainty of  $R$  in the low  $Q^2$ , low  $x$  region. For the intermediate values of  $Q^2$  this model coincides with SLAC parametrization, whereas for lower values of  $Q^2$ , outside the range of the validity of this parametrization, it quickly vanishes as  $Q^2 \rightarrow 0$ .

In fig. 10 we show extrapolation of our model calculations for  $R$  up to  $x = 0.1$  and confront this extrapolation with the available data. Although this value of  $x$  is already too large for our model to be applicable one can see that it predicts reasonably well the magnitude and the  $Q^2$  dependence of  $R$ . The agreement with experimental results improves if we renormalise the phenomenological parameter  $A$  by a factor of 1.2 – 1.5 .

## 5 Summary and conclusions

In this paper we have presented possible parametrization of the longitudinal structure function  $F_L$  and of the ratio  $R(x, Q^2)$  in the region of low  $Q^2$  and low  $x$ . We based this parametrization on the photon-gluon fusion mechanism suitably extrapolated into the region of low  $Q^2$ . The extrapolation respected the kinematical constraint  $F_L \sim Q^4$  in the limit  $Q^2 \rightarrow 0$ . We have also

included a separate contribution from the higher twist term which corresponds to soft pomeron exchange. Its coupling to external virtual photons was assumed to be proportional to that of the (on-shell) gluons but with limited transverse momentum of the quarks (antiquarks) within the quark box (and crossed-box) diagram, eq. (29). We have also calculated the ratio  $R(x, Q^2) = F_L(x, Q^2)/F_T(x, Q^2)$  utilising the parametrization of the structure function  $F_2(x, Q^2)$  from ref. [32]. Our results show that the structure function  $F_L$  and the ratio  $R$  become negligibly small in the region of low  $Q^2$  ( $Q^2 < 0.1$  GeV $^2$  or so). The ratio  $R(x, Q^2)$  turns out to be essentially independent of  $x$  in the low  $x$ , low  $Q^2$  region. We have also noticed that in this region the magnitude of both  $F_L$  and of the ratio  $R$  is relatively insensitive to the variations of the input parton distributions. We believe that our results will also be useful in the radiative corrections procedure as well as in extracting the magnitude of the spin dependent structure functions from the experimental data. The codes for calculating  $R(x, Q^2)$  are available upon request from a.m.stasto@durham.ac.uk.

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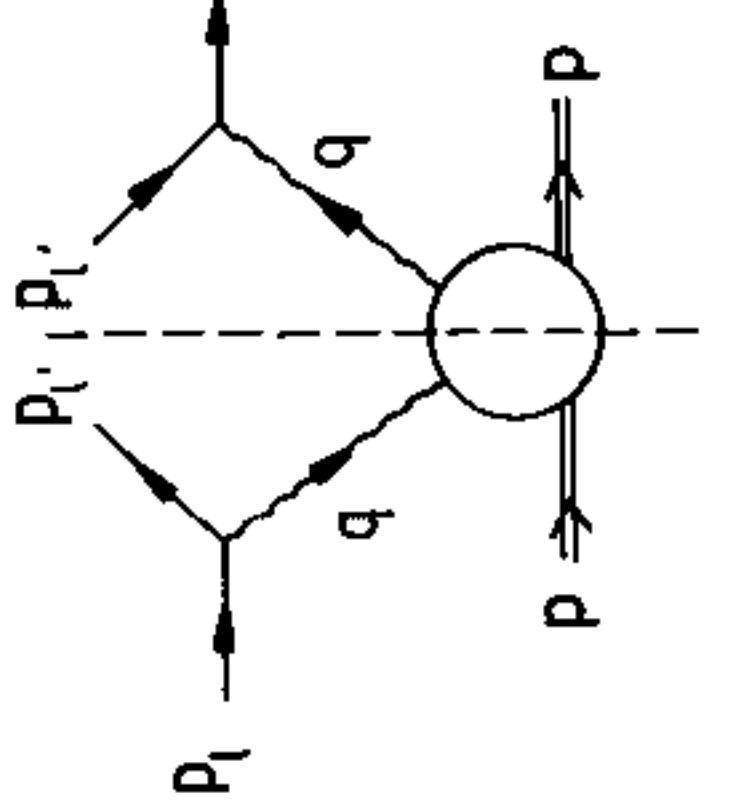
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## Figure captions

1. Kinematics of inelastic charged lepton–proton scattering in the one–photon–exchange approximation and its relation through the optical theorem to Compton scattering for the virtual photon;  $p_l$ ,  $p$  and  $q$  denote the four momenta of the incident lepton, proton and virtual photon respectively.
2. Compilation of measurements of  $R$ ; data come from SLAC E140X [9], SLAC E140 and the global analysis of earlier SLAC experiments [10], CDHSW [11], BCDMS [12] and EMC [13] experiments. Errors show the statistical and systematic uncertainties added in quadrature but in case of SLAC E140X and the global analysis of the SLAC data they do not include the systematic error due to radiative corrections (equal to approximately 0.012 and 0.025 respectively). The curve is a fit to all of the data (excluding E140X), [10, 14]. The figure is taken from [15].
3. The diagrammatic representation of the photon–gluon fusion mechanism and of the  $k_T$  factorization formula (12).
4. Longitudinal structure function  $F_L$  calculated as a function of  $x$  for different values of  $Q^2$  assuming the MRS(A) and GRV gluon distributions for both the on- and off-shell approaches.
5. As fig.4 but  $F_L$  is calculated as a function of  $Q^2$  for different values of  $x$ .
6. As fig.4 but results include a contribution from higher twist.
7. As fig.5 but results include a contribution from higher twist.

8.  $R(x, Q^2)$  plotted as a function of  $x$  for different values of  $Q^2$  and containing a contribution from higher twist.
9. As fig.8 but  $R$  is plotted as a function of  $Q^2$  for different values of  $x$ .
10. Comparison of the model calculation for  $R$ , extrapolated to  $x=0.1$ , with the data. The higher twist is taken into account in the calculations. See fig. 2 for a description of the data.

$$-q^2 = Q^2 \quad x = \frac{Q^2}{2pq}$$



=

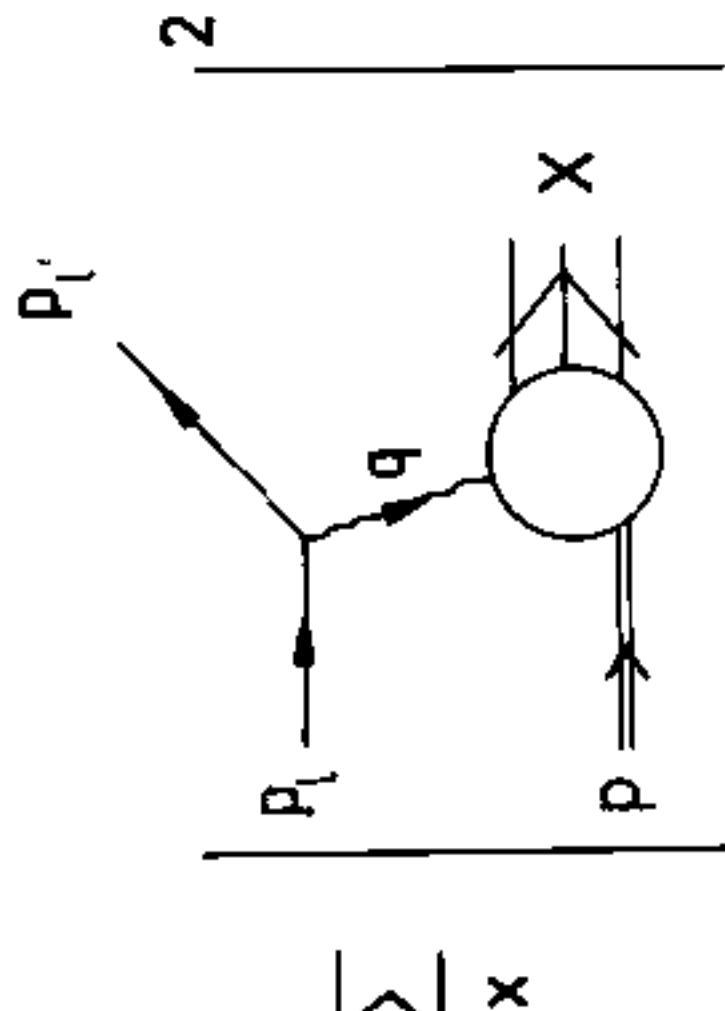
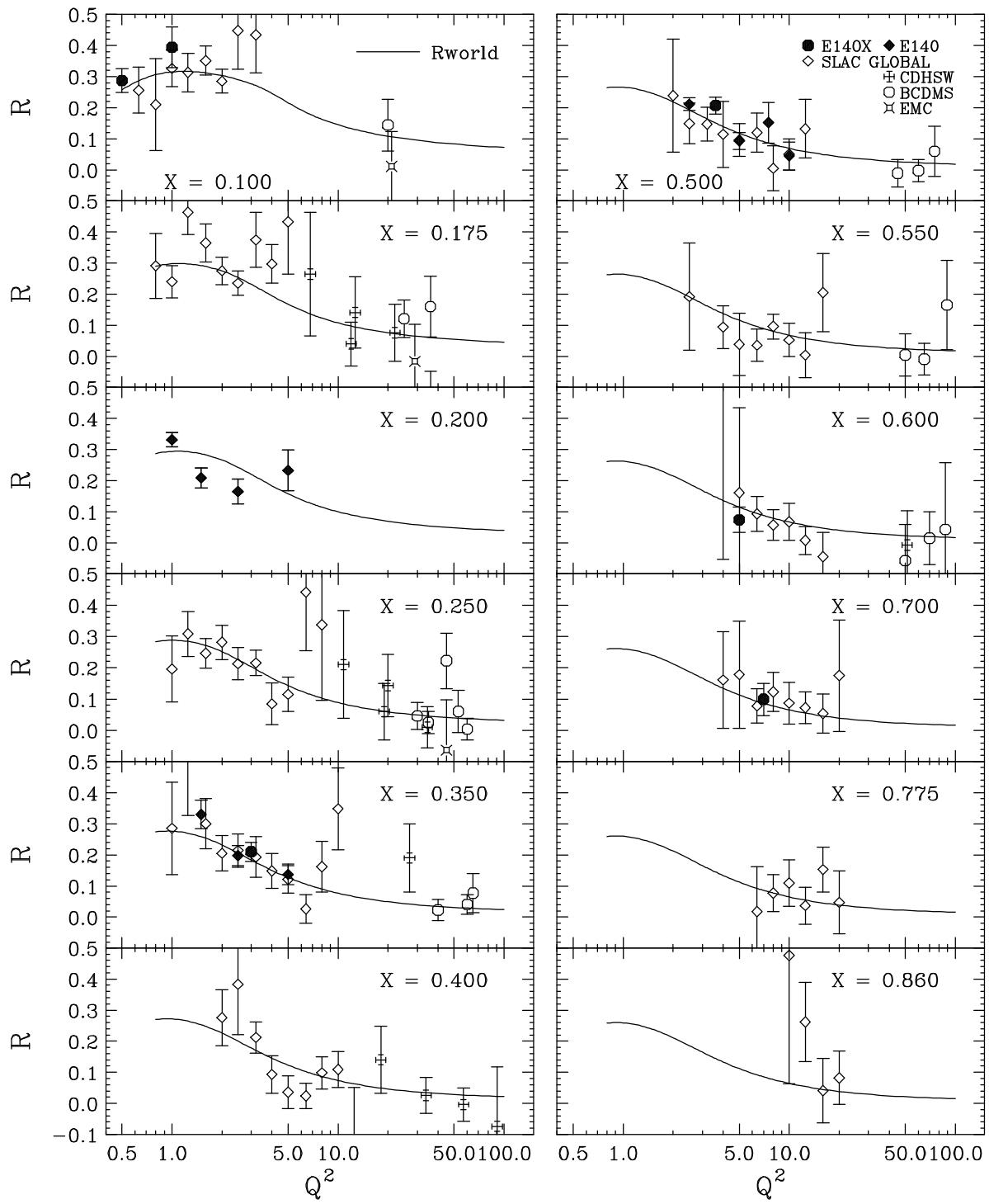
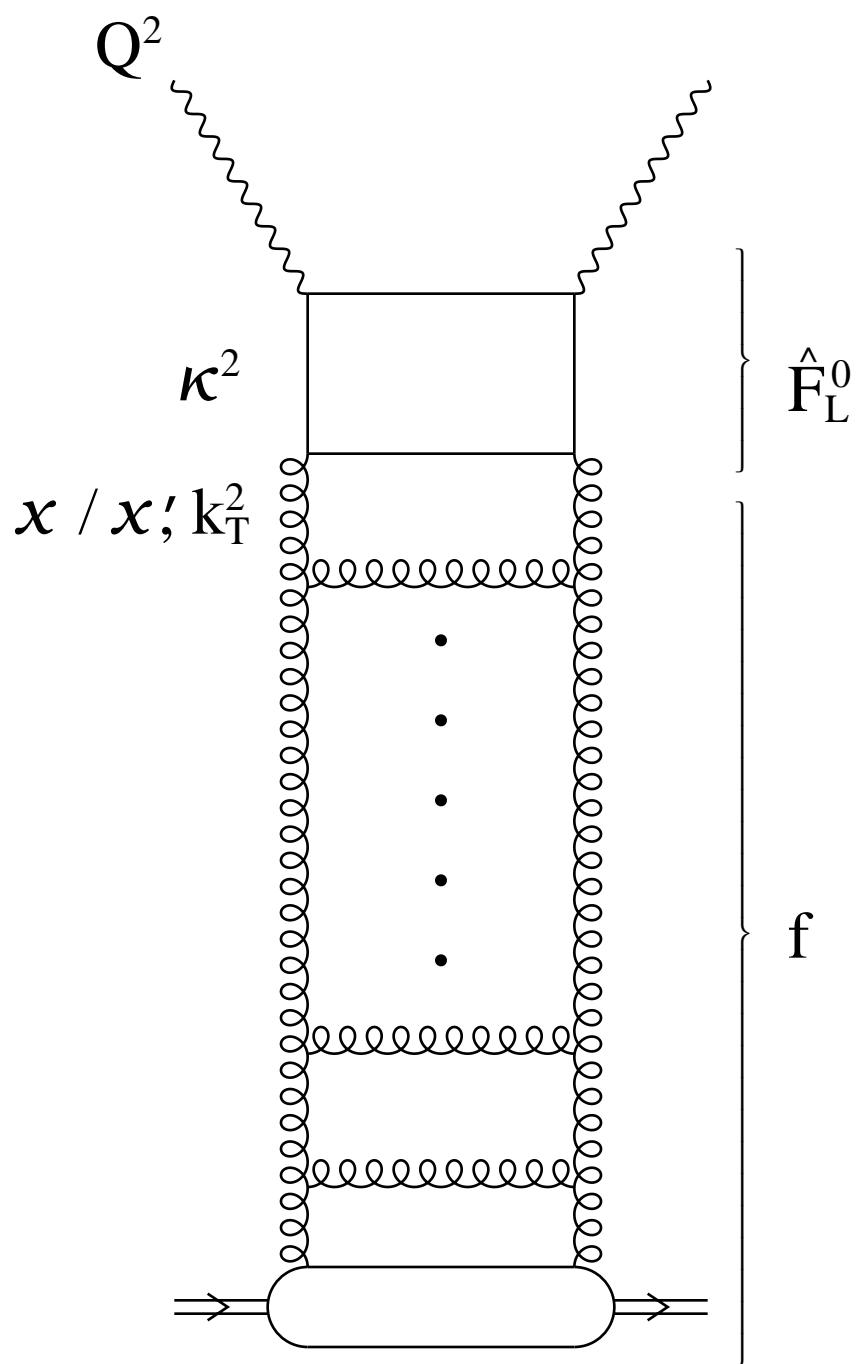


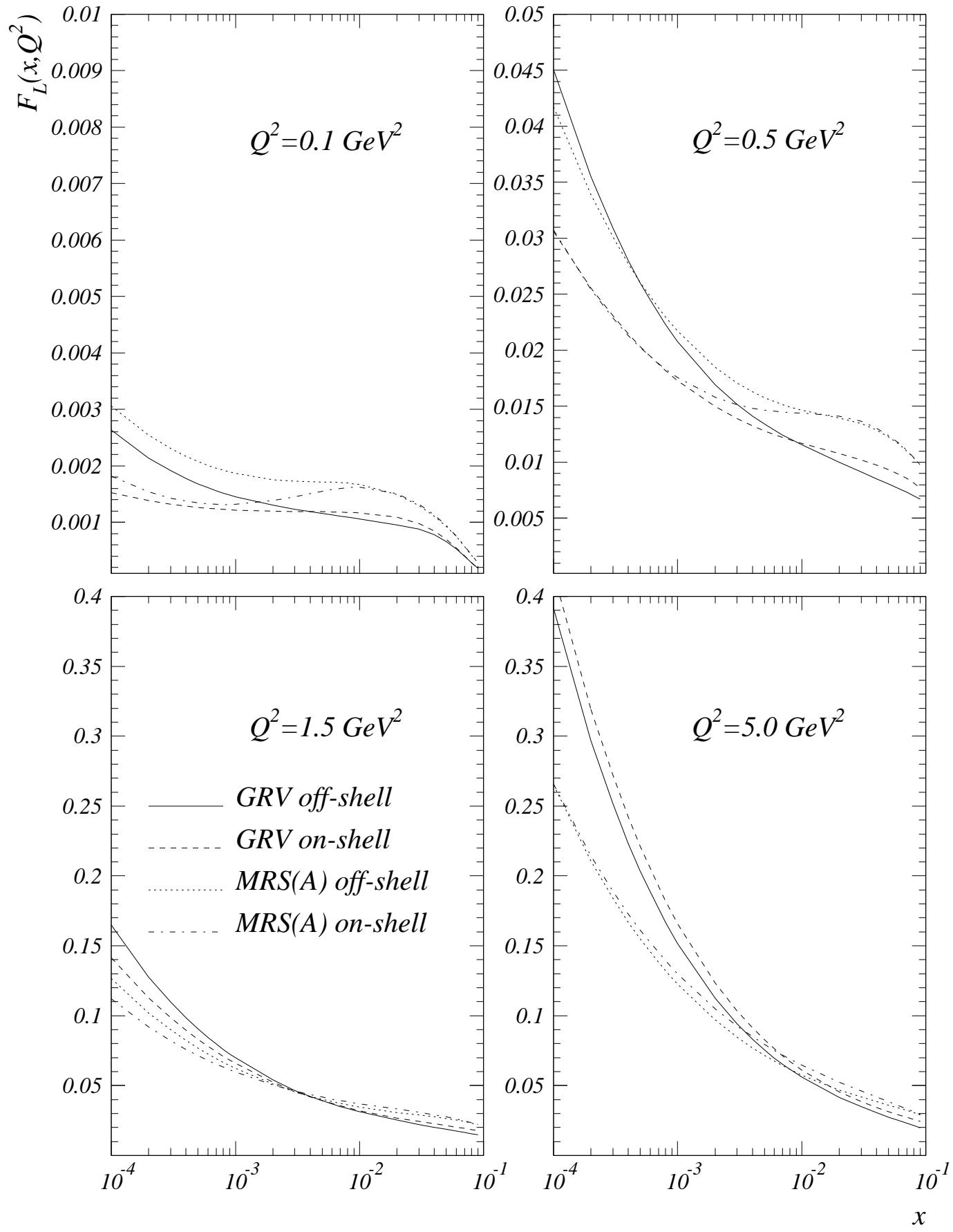
Fig. 1



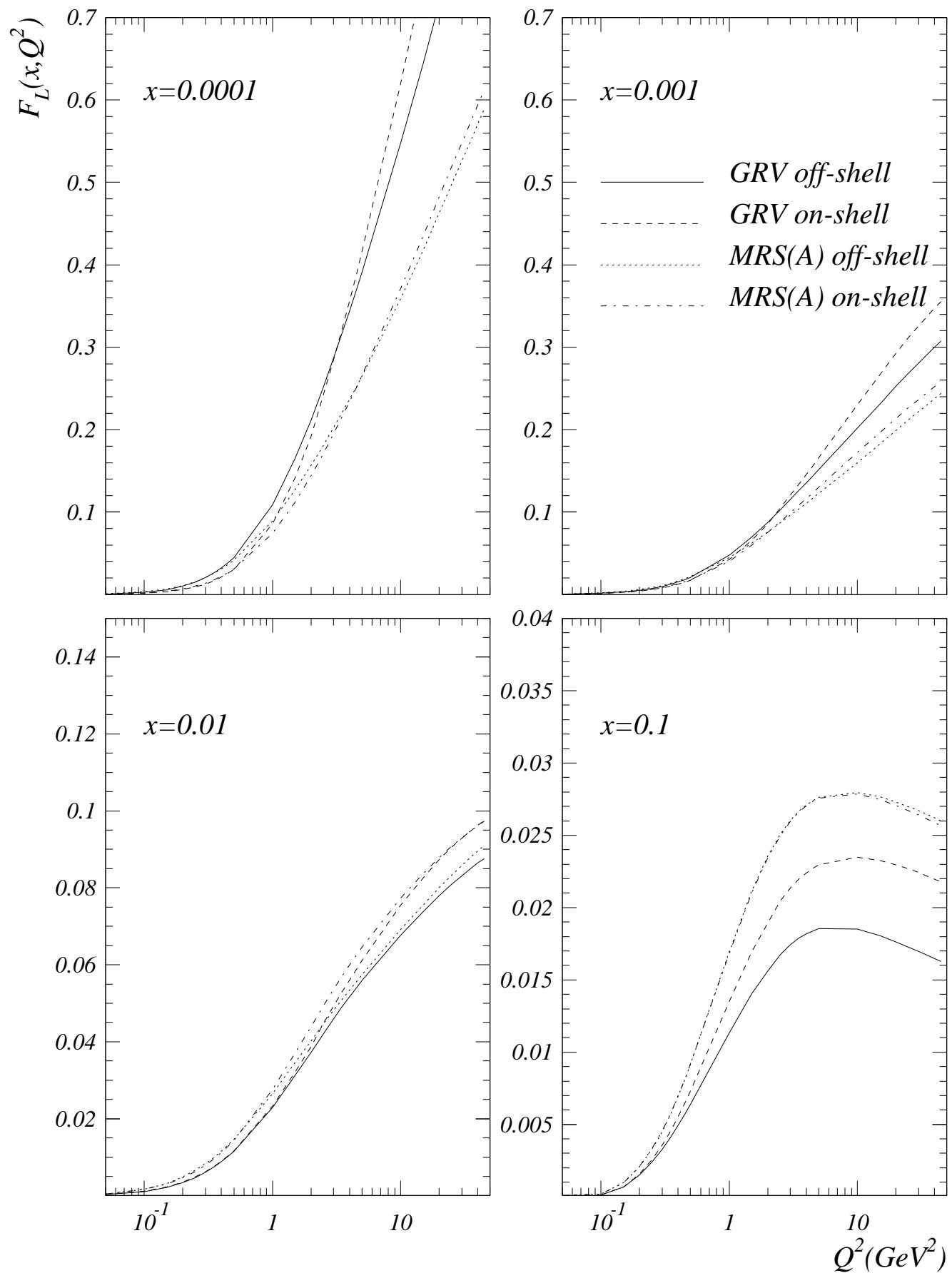
**Fig.2**



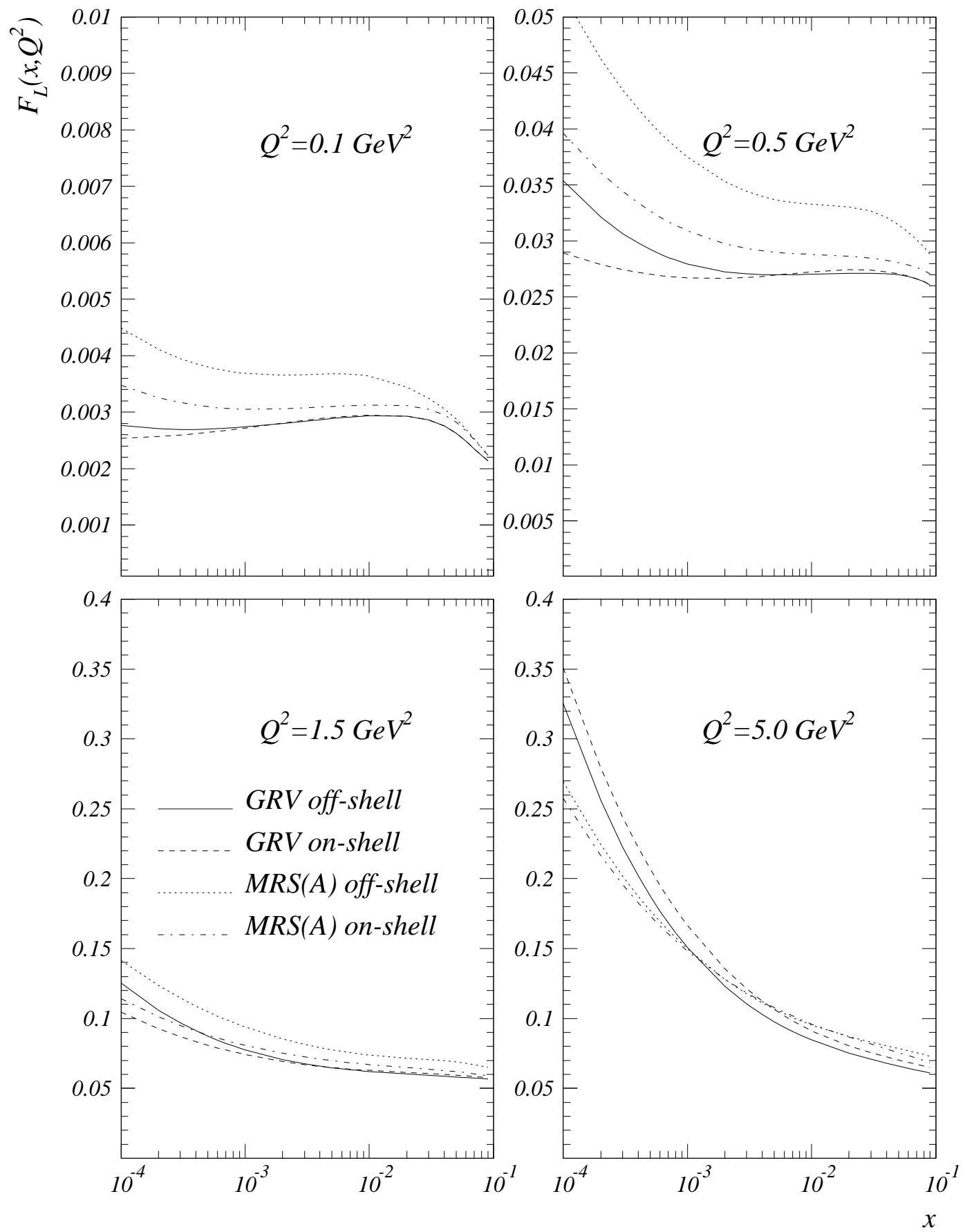
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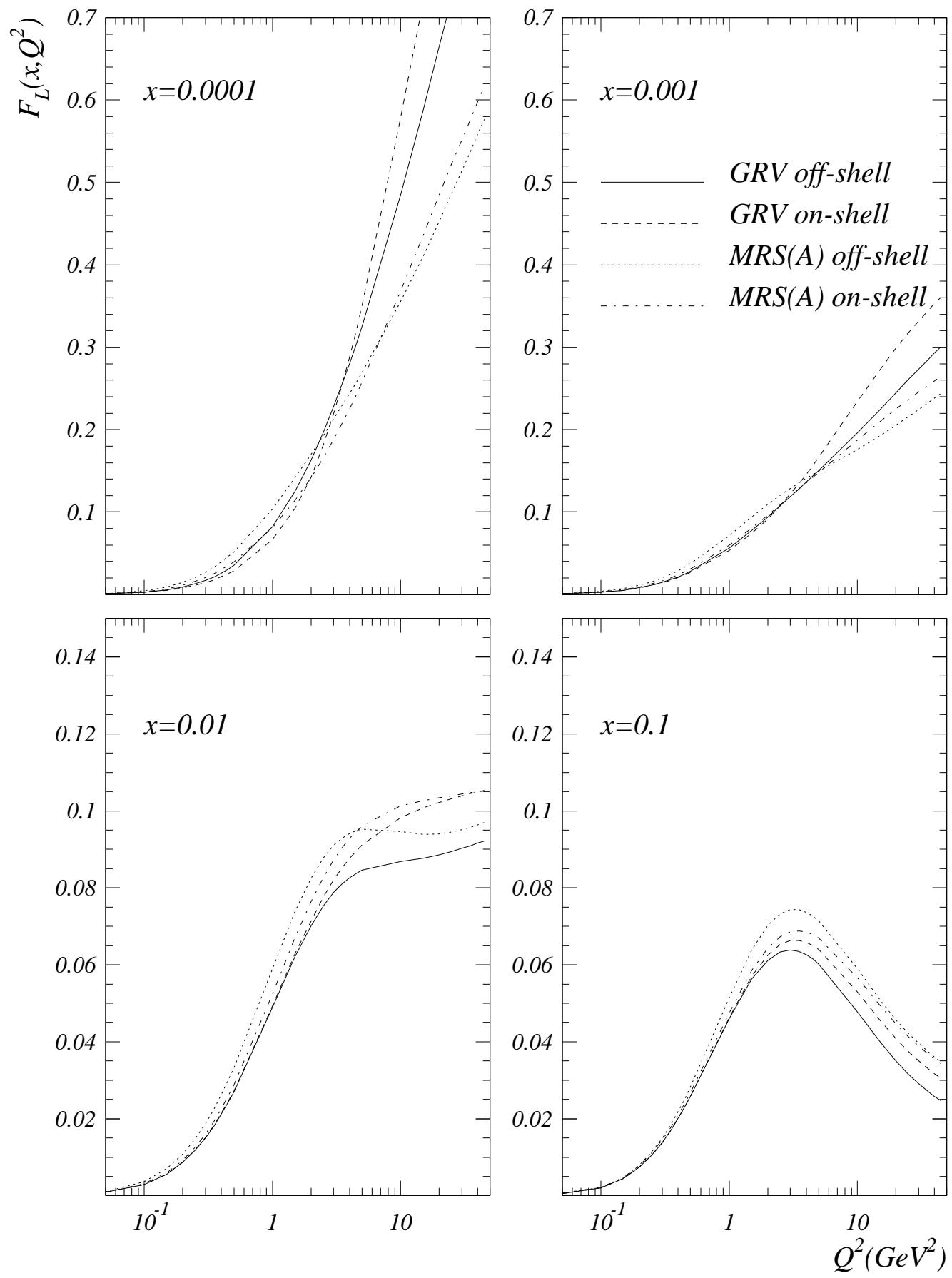
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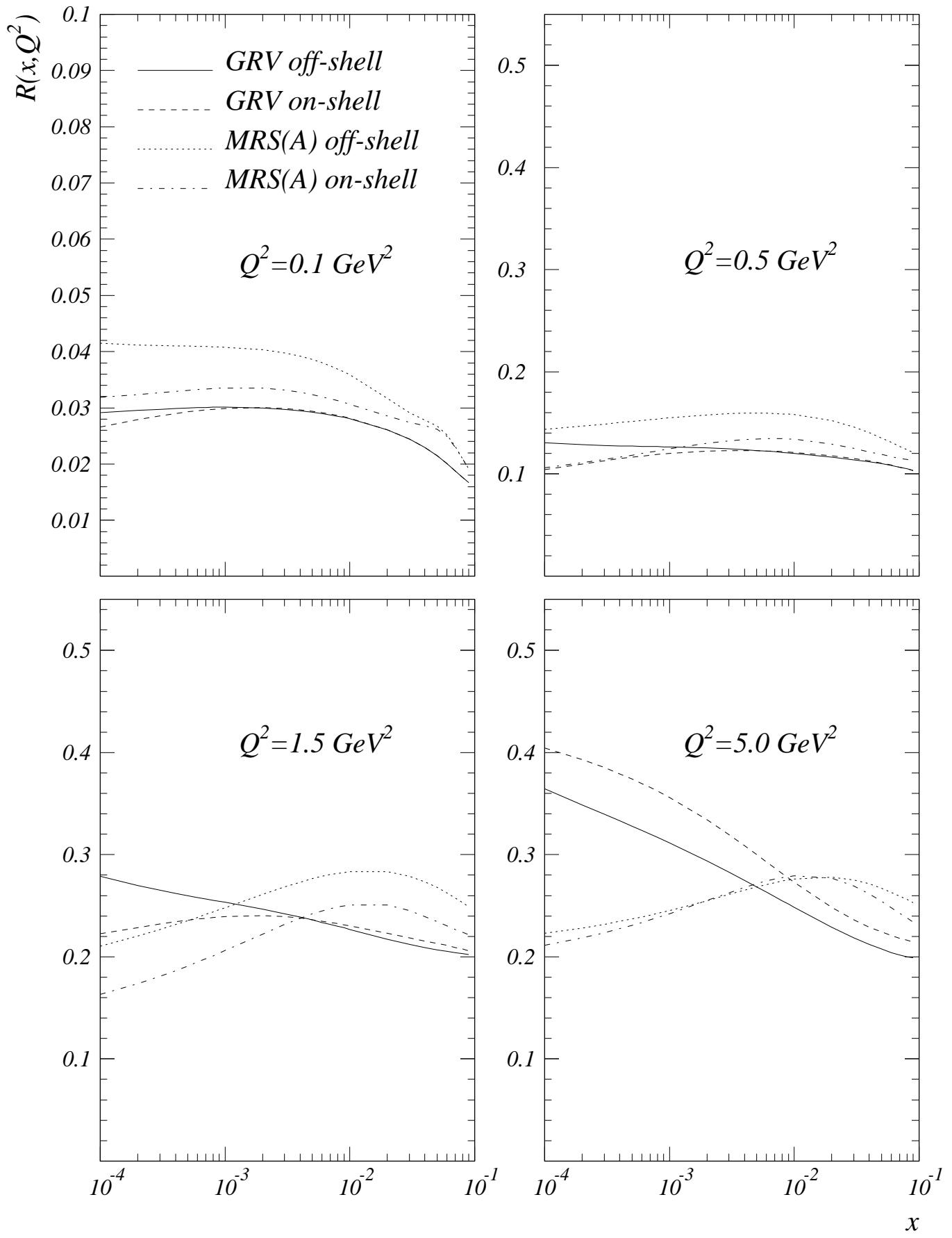
**Fig.5**



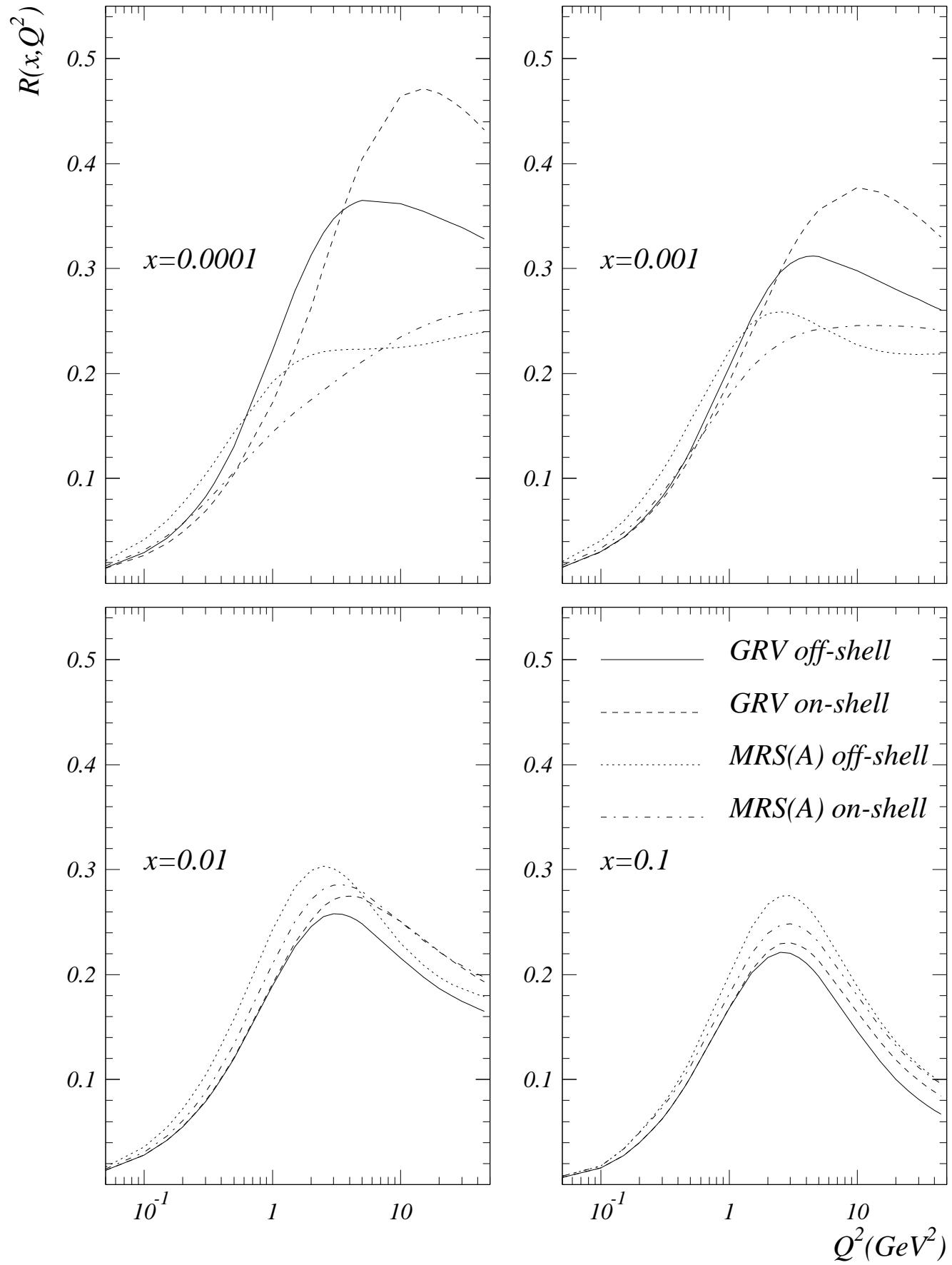
**Fig.6**



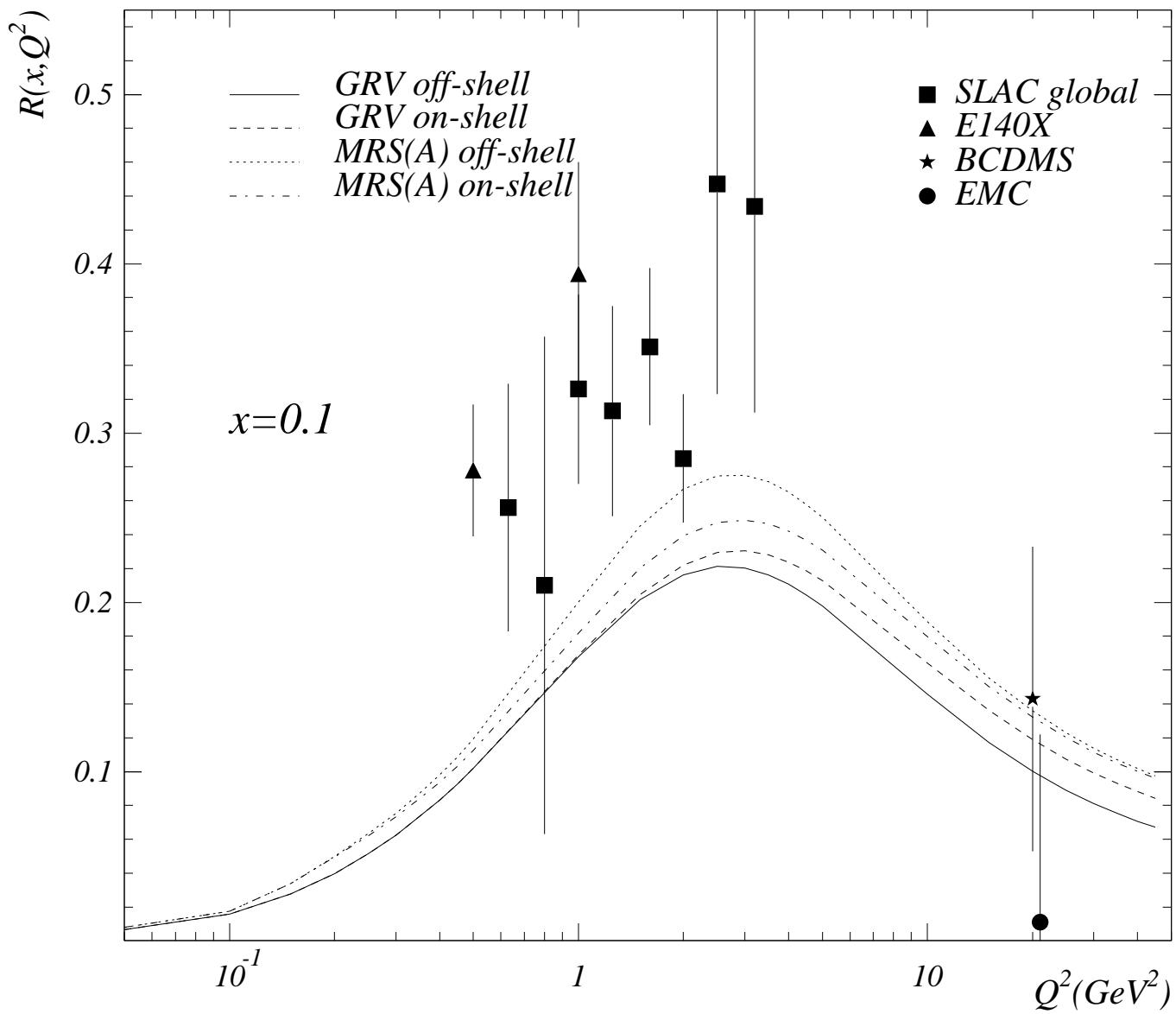
**Fig.7**



**Fig.8**



**Fig.9**



**Fig.10**